Static Parameters of Hadrons and Quantum Groups

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Abstract

We study the static properties of hadrons, assuming quantum group symmetry. We calculate the magnetic moment, axial form factor and A-symmetry, using $SU_q(2)$ and $SU_q(3)$ quantum groups. The results are fitted with experimental data, giving an interval of $0.9 < q < 1.1$. Some of the implications for the deformation parameter are discussed.

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1 Introduction

Despite rather extended study of quantum groups as mathematical objects[1-7], little has been done in the way of connecting them with observable phenomena, but some papers in this direction have appeared [8-11]. In such theories, the deformation parameter q is a function of a dimensionless combination of constants, so that in the limit of $q \rightarrow 1$, we recover the undeformed theories, this may for example correspond to the low energy limit. For instance, Newtonian mechanics can be thought of as the low velocity limit ($v/c \rightarrow 0$) of the relativistic mechanics. In this paper we use quantum groups in deforming the flavour symmetry of hadrons, with the help of the standard q-deformation [1] of $SU_q(2)$ and $SU_q(3)$. Knowing that flavour symmetry is not an exact symmetry of nature, we chose to deform the flavour group rather than the other symmetry groups of hadrons. However, as we shall see later, it becomes clear that deformation of either spin or colour symmetry is unavoidable. We placed hadrons in representations of quantum groups so that in the limit $q \rightarrow 1$, the standard decuplet, octet and singlet representations are recoverd.

The price of using quantum groups as symmetry group of fundamental particles is to abandon the notion of permutation symmetry of fermions and bosons. The representation theory of the classical groups is closely linked with the group of permutation of n objects. Therefore, the concepts of anti-symmetry of wavefunctions under the permutation of pairs of fermions and symmetry under the permutation of pairs of bosons,can be incorporated naturally, using anti-symmetric and symmetric tensor representations of the classical groups. This device is not natural within the representation theory of quantum groups. Here the permutation group is replaced by its deformed version; the Braid group [14]. The braid

group B_n deals with the permutation of n strings, and in general can be very complex. The relevant braid group that we are concerned with in this paper, satisfies a quadratic relationship (Skein relation):

$$B_{ij}^2 + (q^2 - 1)B_{ij} = q^2 \tag{1.1}$$

where B_{ij}, tangles two strands i and j. This, replaces the simple quadratic relationship of permuting two objects i and j in a row of n objects; $P_{ij}^2 = 1$. It is clear that the relation (1.1) reduces to the permutation relation in the limit $q \to 1$. If we postulate that the physical states are eigenstates of B rather than P, then we will have no choice other than letting bosons correspond to an eigenstate with the unit eigenvalue and fermions to the eigenstate with the eigenvalue equal to $-q^2$. Here we face a problem, that the octet states are not the eigenstates of the braid matrix; the operation of B on these states will not leave them invariant. Thus the deformation of flavour alone can not be permitted, so we conclude that deforming another subspace such as spin is necessary [12].

The paper is organised as follows: in Sec.2 we give a brief introduction to quantum groups, construct the q-deformed states of the flavour space, and then together with the q-deformed spin space, we discuss the effects of the braid group. In the Sec.3, following the standard procedures, we calculate the static parameters of hadrons as functions of q, fitting these calculations with the observed data, we find an interval for q.

2 Quantum Groups

Every quantum group corresponds to a solution of the Yang- Baxter equation, and to every solution we can correspond an integrable statistical model [13]. There are in fact mathemat-

ical relationships between integrable models and quantum groups [1-2]. Here we shall only give a brief discussion of the mathematical structure of the standard deformation of $SU_q(3)$, which will suffice for our calculations. The commutation relations of $SU_q(3)$ algebra are:

$$\begin{aligned}
[H_i, H_j] &= 0 \\
[H_i, X_j^{\pm}] &= a_{ij} X_j^{\pm} \\
[X_i^{+}, X_j^{-}] &= \delta_{ij} \tfrac{q^{H_i} - q^{-H_i}}{q - q^{-1}} \qquad\qquad i, j = 1, 2
\end{aligned} \tag{2.2}$$

where H_i, $X_i^{\pm}$ and a_{ij} are Cartan generators, ladder operators and Cartan matrix elements respectively.

This algebra is characterized by Hopf algebra structure; the coproduct Δ, coidentity ϵ and antipode S are defined as [3]:

$$\begin{aligned}
\Delta H_i &= H_i \otimes 1 + 1 \otimes H_i \\
\Delta X_i^{\pm} &= q^{-H_i/2} \otimes X_i^{\pm} + X_i^{\pm} \otimes q^{H_i/2} \\
(\Delta \otimes 1)\Delta X_i^{\pm} &= q^{-H_i/2} \otimes q^{-H_i/2} \otimes X_i^{\pm} + q^{-H_i/2} \otimes X_i^{\pm} \otimes q^{H_i/2} \\
&\quad + X_i^{\pm} \otimes q^{H_i/2} \otimes q^{H_i/2},
\end{aligned} \tag{2.3}$$

$$\begin{array}{lll}
\epsilon(1) = 1 & \epsilon(H_i) = 0 & \epsilon(X_i) = 0 \\
S(1) = 1 & S(H_i) = -H_i & S(X_i^{\pm}) = -q^{H_i/2} X^{\pm} q^{-H_i/2}.
\end{array} \tag{2.4}$$

Choosing a_{ij} accordingly, one obtains the standard deformation of the relevant Lie algebra. Representations of quantum groups can be constructed by similar routes to the undeformed version. We are concerned here with the fundamental representations.

For $SU_q(2)$ algebra the matrix representations of H and $X^{\pm}$ are:

$$\begin{gathered}
H = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
X^{+} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad\qquad X^{-} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}
\end{gathered} \tag{2.5}$$

and for $SU(3)_q$:

$$
\begin{aligned}
H_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad H_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \\
X_1^+ &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad X_1^- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
X_2^+ &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad X_2^- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.
\end{aligned} \tag{2.6}
$$

Hadrons are made of three quarks each of which can occur in three types of flavours (up,down,strange),

$$3 \otimes 3 \otimes 3 = 3 \otimes (6 \oplus \bar{3}) = 10_S \oplus 8_{MS} \otimes 8_{MA} \oplus 1_A. \tag{2.7}$$

So there are 27 possible combinations which can be separated into totally symmetric (S), mixed-symmetric (MS), mixed-antisymmetric (MA) and totally antisymmetric (A) states. By applying the ladder operators $X_i^{\pm}$ to the highest weight of these 27 states(uuu) with the use of the coproduct (2.3), we construct the 27 q-deformed states of flavour space. Here we only give an example for each S, MS, MA and A states in flavour space. Now consider the statistics of these particles. The permutation operator is defined as:

$$
\begin{aligned}
P_{12}|a>|b> &= |b>|a> \\
P_{12}^2 &= 1
\end{aligned} \tag{2.8}
$$

where $|a>$, $|b>$ are the states of 2 identical particles and the eigenvalues are ± 1,for S and A states. The eigenstates of P_{12} are:

$$\begin{aligned} |ab>^{+} &= \tfrac{1}{\sqrt{2}}(|a>|b>+|b>|a>) \\ |ab>^{-} &= \tfrac{1}{\sqrt{2}}(|a>|b>-|b>|a>). \end{aligned} \qquad (2.9)$$

The system containing N identical particles is either totally symmetric under the interchange of any pair; the eigenvalue of P_{12} for them is $+1$ (Bosons), or totally antisymmetric, with the eigenvalue -1 (Fermions),

$$\begin{aligned} P_{ij}|a_1...a_i...a_j...> &= (+1)|a_1...a_j...a_i...> \qquad for \quad Bosons \\ P_{ij}|a_1...a_i...a_j...> &= (-1)|a_1...a_j...a_i...> \qquad for \quad Fermions. \end{aligned} \qquad (2.10)$$

However all of the q-deformed flavour states are not eigenstates of P, in fact they are eigenstates of the braid operator B. To maintain a meaningfull deformation there does not seem to be a choice other than letting particle states to be eigenstates of the braid operator. This changes our notion of fermions and bosons.

Braids are made of n points on a line that are connected by n strings to n point on another parallel line (Fig. 1.a). With the operation b_i, $i = 1, 2, ..., n-1$, we cross over two neighbouring strands i and $i+1$ (Fig. 1.b) [14]. The simple braid operation satisfies,

$$\begin{aligned} b_i b_{i+1} b_i &= b_{i+1} b_i b_{i+1} \\ b_i b_j &= b_j b_i \qquad |i-j| \geq 2. \end{aligned} \qquad (2.11)$$

The multiplication of braid operators is the placing of braids such as Fig. 1.b., thus the identity is the trivial braid of Fig. 1.a. The braid group B_n, is composed of all the tangling moves possible on this structure. In general the group B_n has infinite size, unless its generators satisfy some polynomial relationship; the Skein relation. The Skein realtion offers a

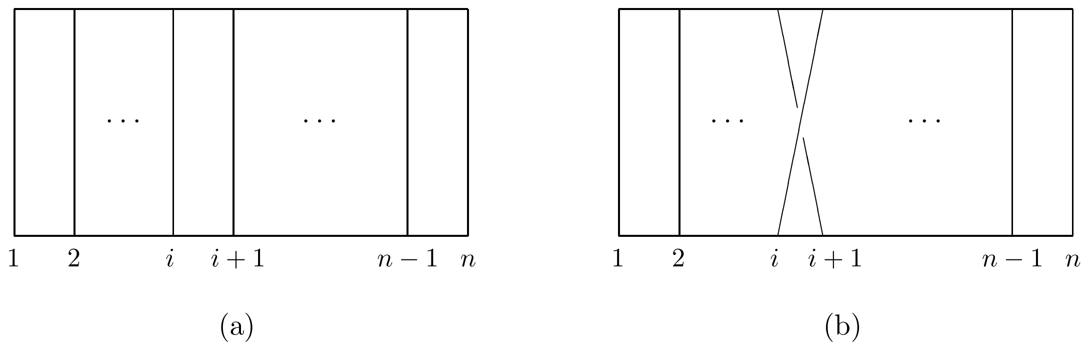

Figure 1: Graphical presentation of $n - braid$ and the operation of b_i on it.

way of untangling a braid. For our case, the Skein relation (1.1) is equivalent to the Fig. 2. With the use of a solution of the Yang-Baxter equation R [13],

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12} \tag{2.12}$$

a braid matrix is defined as:

$$B = P^{-1}R \tag{2.13}$$

where P is the permutation matrix of the equation (2.9). So the matrix of our braid is:

$$B \;=\; \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-q^2 & q & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.14}$$

with the eigenvalues 1 , $-q^2$ for S and A states respectively, and now the braid-permutation

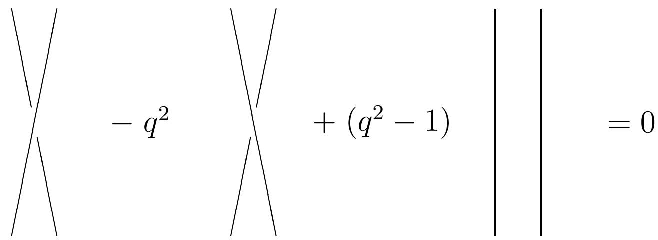

Figure 2: The Skein Relation of (1.1)

matrix on a system of two quarks, acts as:

$$
B \begin{pmatrix} uu \\ ud \\ du \\ dd \end{pmatrix} = \begin{pmatrix} uu \\ (1-q^2)ud + qdu \\ qud \\ dd \end{pmatrix}
$$

$$
B \begin{pmatrix} uu \\ us \\ su \\ ss \end{pmatrix} = \begin{pmatrix} uu \\ (1-q^2)us + qsu \\ qus \\ ss \end{pmatrix} \tag{2.15}
$$

$$
B \begin{pmatrix} dd \\ ds \\ sd \\ ss \end{pmatrix} = \begin{pmatrix} dd \\ (1-q^2)ds + qsd \\ qds \\ ss \end{pmatrix} .
$$

We observe that the exchange of particles is unusual, but when $q \to 1$ we recover the old results. Fortunately, the tower of braided states due to the Skein relation (1.1), does not

extend indefinitely.

Now for example, the symmetric state of $|\Delta^+>$ under the exchange of the first two quarks becomes:

$$\begin{aligned} B_{12}|\Delta^+> &= B_{12}(\tfrac{1}{\sqrt{1+q^2+q^4}})(uud+qudu+q^2duu) \\ &= (\tfrac{1}{\sqrt{1+q^2+q^4}})[uud+q(1-q^2)udu+q^2duu+q^3udu] \\ &= (+1)(\tfrac{1}{\sqrt{1+q^2+q^4}})(uud+qudu+q^2duu) \\ &= (+1)|\Delta^+> \end{aligned} \tag{2.16}$$

and in the only anti-symmetric state; $|\Lambda_1^o>$ this exchange of the first two quarks gives:

$$\begin{aligned} B_{12}|\Lambda_1^o> &= B_{12}(\tfrac{1}{\sqrt{(1+q^2)(1+q^2+q^4)}})[(sdu-qsud)+(q^2usd-qdsu)+(q^2dus-q^3uds)] \\ &= \tfrac{1}{\sqrt{(1+q^2)(1+q^2+q^4)}}[qdsu-q^2usd+q^2(1-q)-q^2sdu \\ &\quad +q^3uds-q^3(1-q^2)uds-q^4dsu] \\ &= (-q^2)(\tfrac{1}{\sqrt{(1+q^2)(1+q^2+q^4)}})[(sdu-qsud)+(q^2usd-qdsu)+(q^2dus-q^3usd)] \\ &= (-q^2)|\Lambda_1^o>\,. \end{aligned} \tag{2.17}$$

Therefore, although the state $|\Delta^+>$ can be interpreted as symmetric, this is not the case for $|\Delta_1^o>$. So we are led to require this unorthodox interpretation of permutation of identical subsystems of a bound state. This is a fundamental change of our notions, but seems inevitable if quantum groups are to find physical relevance. This led us to proceed with this proviso in mind and look at the physical consequences of this construct. Now we see that a hadron state like proton; $|p>$,in which the state is constructed by a combination of MS and MA states, can not be an eigenstate of braid matrix. This problem leads us to conclude that the deformation of only one subspace is not enough, that another subspace has to be deformed. Spin space seems to be the best choice among the others. Tables 1, 2 and 3 show

the deformed states for spin and flavour states.

Finally the permutaion matrix is defined as follows:

$$\mathcal{P} = P_{space} \otimes B_{spin} \otimes B_{flavour} \otimes P_{colour} \tag{2.18}$$

where the indices define the space on which these operators act. Here B_{spin} has the eigenvalues of 1 and $-q^{-2}$ for the S and A states respectively. So again we will have ± 1 as the eigenvalues of the total permutation matrix. We also note that $\mathcal{P}$ is a unitary matrix, and an expectation value of an obsevable like A for a particle with the state $|a>$, can have a unitary transfomation under $\mathcal{P}$:

$$< a|\mathcal{P}^{\dagger} A \mathcal{P}|a> = < a|A|a> . \tag{2.19}$$

3 Phenomenological Calculations

The wave-function of a hadron has to be antisymmetric because, quarks are fermions and the wave-function is written as:

$$|\Psi> = |Space, Spin> |Flavour> |Colour> . \tag{3.20}$$

The spape-spin wave function can be separated if the model is non-relativistic [15,16]. It's usual to take the colour state totally antisymmetric, so the rest should be totally symmetric. For hadrons we have to consider a combination of MS and MA states which finally is totally symmetric:

$$|\Psi> = \frac{1}{\sqrt{2}}(\phi_{MS}.\psi_{MS} + \phi_{MA}.\psi_{MA})|Colour>, \tag{3.21}$$

where ϕ's and ψ's are the wave functions of space-spin and flavour respectively.

The static parameters that we have calculated are the magnetic moment μ [16], axial form factor g_A [16] and Bjorken sum-rule $g_1^p - g_1^n$ [15].

The static parameters are:

$$\begin{aligned} g_A &= \Delta u - \Delta d \\ & \quad or \\ g_A &= \textstyle\sum_{i=1}^{3} \sigma_i^3 \tau_i^3 \\ g_1^p - g_1^n &= \tfrac{1}{2}(\tfrac{4}{9}\Delta u + \tfrac{1}{9}\Delta d) \end{aligned} \tag{3.22}$$

and

$$\mu = \sum_{i=1}^{3} O_i(x_i, \sigma_i) e_i \tag{3.23}$$

where

$$\begin{aligned} <\uparrow |O_i| \uparrow> &= a \\ <\downarrow |O_i| \downarrow> &= -a \end{aligned} \tag{3.24}$$

in which i sums over the three quarks, σ_i^3 and τ_i^3 are the spin and the isospin in the z direction of each of them respectively and μ is the magnetic moment of the hadron with "a" as a number depending on the model we use. Here $\Delta u(d) = u \uparrow (d \uparrow) - u \downarrow (d \downarrow)$ is the quark asymmetry, $u \uparrow (\downarrow)$ and $d \uparrow (\downarrow)$ are the probability of finding a u (d) quark with spin polarized (unpolarized)to the spin of proton. We have calculated these parameters for hadrons by using the wave function (3.21) and the states of the tables 1 and 3. In theory, for calculating the magnetic moments, it is convenient to find them with respect to magnetic moment of neutron. We have compared the results with the experimental data [17,18] and found two Least Square Error(LSE) functions for magnetic moments and sum rules as follows

respectively:

$$
\begin{aligned}
LSE(MM) &= \sum_{i=1}^{5}[(\mu_i(q)/\mu(q)_{neutron} - \mu_i^{exp}/\mu_{neutron}^{exp})/\Delta(\mu_i^{exp}/\mu_{neutron}^{exp})]^2 \\
LSE(SR) &= \sum_{i=1}^{4}[(g_i(q) - g_i^{exp})/\Delta(g_i^{exp})]^2
\end{aligned}
\tag{3.25}
$$

where i sums over the particles and Δ's are the experimental errors. For the magnetic moments we have only 5 fuctions because, the magnetic moment found for Ξ^o is the same as of the n and the Σ^o particle is an unstable one.

The experimental results for magnetic moments are more acurate than the sum rules, therefore we have only used the results of LSE(MM). The curve is shown at the end of the paper. As it shows, it has two minimums in $q = 0.916, 1.0915$.

Now we give an example of how we found the static parameters in functions of q, the wave function of proton is:

$$
\begin{aligned}
|P\uparrow> &= \tfrac{1}{(1+q^2+q^4)\sqrt{2}}[q(1+q^2)u\uparrow d\downarrow u\uparrow - q^4 u\downarrow d\uparrow u\uparrow - d\uparrow u\downarrow u\uparrow \\
&\quad + q(1+q^2)d\downarrow u\uparrow u\uparrow - u\uparrow d\uparrow u\downarrow - qd\uparrow u\uparrow u\downarrow + q(1+q^2)u\uparrow u\uparrow d\downarrow \\
&\quad - qu\uparrow u\downarrow d\uparrow - q^3 u\downarrow u\uparrow d\uparrow]
\end{aligned}
\tag{3.26}
$$

Here the $\uparrow$,$\downarrow$ stands for spin up or down of the particle. Next we calculate the expectation value of the magnetic moment operator giving the following result:

$$
< P\uparrow|\mu|P\uparrow>= (\frac{e}{2m})[\frac{7q^2+15q^4+7q^6-q^8-1}{3(1+q^2+q^4)^2}] \tag{3.27}
$$

m is the mass of quark.

The curious thing is that the expectation function of g_A for the proton is different from

neutron,

$$\begin{aligned}
\Delta u &= \tfrac{3q^2(1+q^2)^2}{(1+q^2+q^4)^2} \\
\Delta d &= \tfrac{q^8-q^6-3q^4-q^2+1}{(1+q^2+q^4)^2} \qquad\qquad \textit{all for proton} \\
g_A &= \tfrac{3q^2(1+q^2)^2-q^8+q^6+q^2+3q^4-1}{(1+q^2+q^4)^2} \\
g_1^p - g_1^n &= \tfrac{11q^2+21q^4+11q^6+q^8+1}{18(1+q^2+q^4)^2}
\end{aligned} \tag{3.28}$$

and

$$\begin{aligned}
\Delta u &= \tfrac{q^2-q^4-1}{1+q^2+q^4} \\
\Delta d &= \tfrac{2(1+q^2)}{1+q^2+q^4} \qquad\qquad \textit{all for neutron} \\
g_A &= \tfrac{3q^4-q^2+3}{1+q^2+q^4} \\
g_1^p - g_1^n &= \tfrac{7q^4+q^2+7}{18(1+q^2+q^4)},
\end{aligned} \tag{3.29}$$

and all in $q = 1$ give the undeformed results.

4 Discussions

The values obtained for q are inverse of eachother, and this shows that as we have expected, the algebra is invariant under the exchange of $q \to q^{-1}$. From the exact symmetry of spin, we also have expected that the values for q should be near 1; $0.9 < q < 1.1$. Note also that although the permutation group was deformed, the concept of fermions and bosons remains intact at the level of hadrons, if we deform spin space too.

Acknowledgments

The author wishes to thank S. Rouhani, M. Sheikholeslami, M. Khorrami and F. Arash for their useful discussions.

Table 1: States of q-deformed 1/2 spin space

States	
S	$\alpha(q^2 \uparrow\uparrow\downarrow + q \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)$
MS	$\gamma[(1+q^2) \uparrow\uparrow\downarrow - (q^3 \uparrow\downarrow\uparrow + q^2 \downarrow\uparrow\uparrow)]$
MA	$\beta(\uparrow\downarrow\uparrow - q \downarrow\uparrow\uparrow)$

Note: $\alpha = \frac{1}{\sqrt{1+q^2+q^4}}$, $\beta = \frac{1}{\sqrt{(1+q^2)}}$ *and* $\gamma = \frac{1}{\sqrt{(1+q^2)(1+q^2+q^4)}}$

are the normalization factors.

Table 2: 10 q-symmetric states of flavour subspace

	Φ_S
$\|\Delta^{++}>$	uuu
$\|\Delta^{+}>$	$\alpha(uud+qudu+q^2duu)$
$\|\Delta^{o}>$	$\alpha(udd+qdud+q^2ddu)$
$\|\Delta^{-}>$	ddd
$\|\Sigma^{*^o}>$	$\gamma[(usd+q^{-1}uds)+(dus+qsud)+(q^2sdu+qdsu)]$
$\|\Sigma^{*^+}>$	$\alpha(uus+qusu+q^2suu)$
$\|\Sigma^{*^-}>$	$\alpha(dds+qdsd+q^2sdd)$
$\|\Xi^{*^-}>$	$\alpha(dss+qsds+q^2ssd)$
$\|\Xi^{*^o}>$	$\alpha(uss+qsus+q^2ssu)$
$\|\Omega^{-}>$	sss

Table 3: 8 Mixed q-symmetric and 8 Mixed q-antisymmetric states of flavour subspace

	Φ_{MS}	Φ_{MA}
$\vert p>$	$\gamma[q(1+q^2)uud-(udu+qduu)]$	$\beta(qudu-duu)$
$\vert n>$	$\gamma[(q^2udd+q^3dud)-(1+q^2)ddu]$	$\beta(qudd-dud)$
$\vert \Sigma^+>$	$\gamma[q(1+q^2)uus-(usu+qsuu)]$	$\beta(qusu-suu)$
$\vert \Sigma^o>$	$\frac{\gamma}{\sqrt{1+q^2}}[(qsud+q^2sdu)+(usd+qdsu)$ $-q(1+q^2)(qdus+uds)]$	$\frac{1}{1+q^2}[(q^2dsu+qusd)-(sud+qsdu)]$
$\vert \Sigma^->$	$\gamma[q(1+q^2)dds-(dsd+qsdd)]$	$\beta(qdsd-sdd)$
$\vert \Lambda^o>$	$\frac{1}{1+q^2}[(-dsu+qusd)+(q^2sud-qsdu)]$	$\delta[(q^{-2}sdu-q^{-1}sud)+(usd-q^{-1}dsu)$ $-(1+q^2)(dus-quds)]$
$\vert \Xi^->$	$\gamma[(q^2dss+q^3sds)-(1+q^2)ssd]$	$\beta(qdss-sds)$
$\vert \Xi^o>$	$q^{-1}\gamma[(q^2uss+q^3sus)-(1+q^2)ssu]$	$\beta(quss-sus)$

Note: $\delta=\frac{q}{(1+q^2)\sqrt{q^4+q^2+q^{-2}}}$ *is the normalization factors.*

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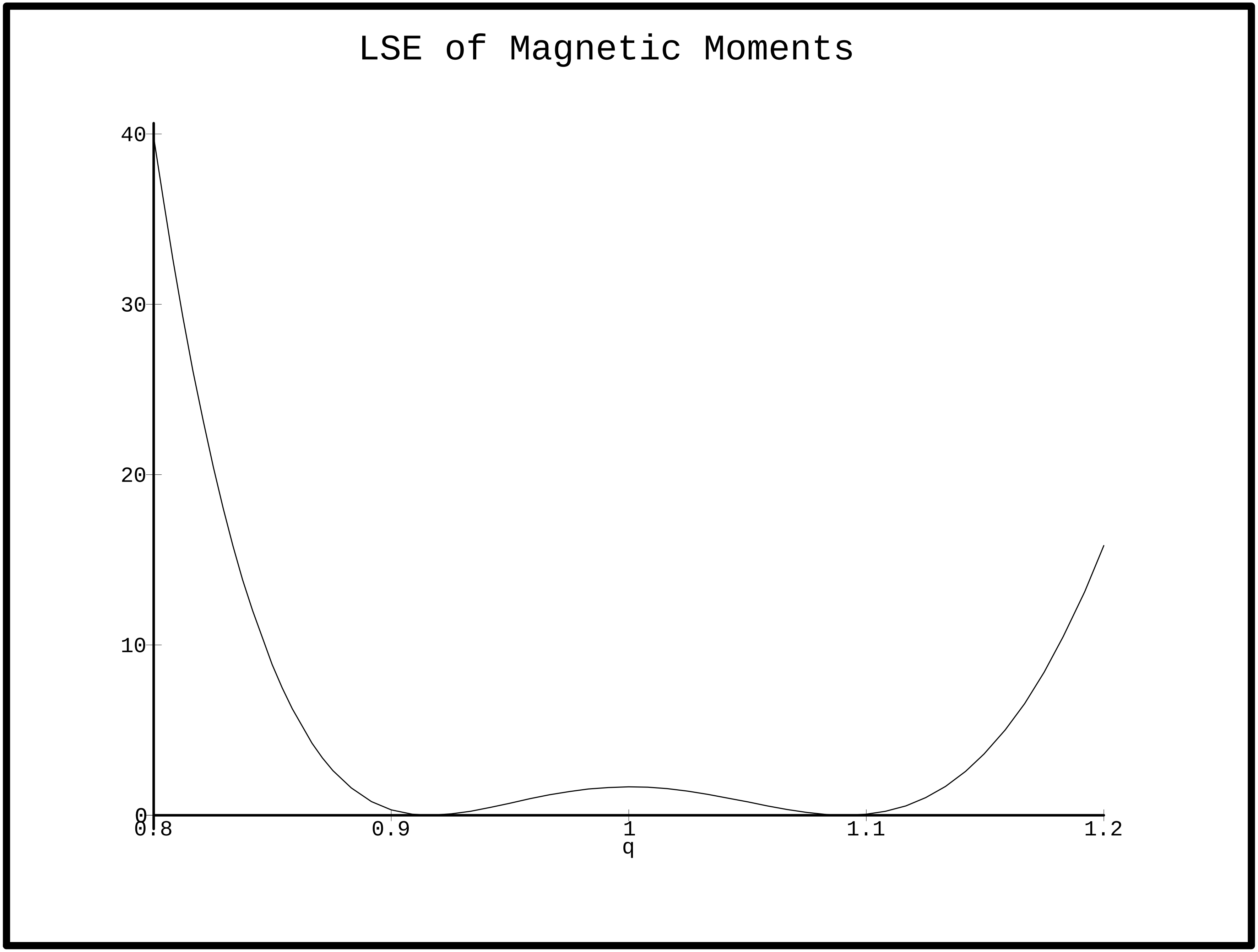
LSE of Magnetic Moments
40
30
20
10
0
0.8
0.9
1
1.1
1.2
q